Solutions Week 2

Q1

a)

In steady state for intermediate steps, their rate should be zero, hence

$$\frac{d[cC]}{dt} = k_c cC - k'_c[cC] - m_c[cC] + m'_c[c^*C] = 0$$

$$\frac{d[c^*C]}{dt} = m_c[cC] - m'_c[c^*C] - l'_c[c^*C] - v[c^*C] = 0$$

Thus we have

$$[cC] = \frac{k_c cC + m'_c[c^*C]}{k'_c + m_c}$$

$$[c^*C] = \frac{m_c[cC]}{m'_c + l'_c + v}$$

Substitute $[cC]$ into $[c^*C]$,

$$[c^*C] = \frac{m_c \dfrac{k_c cC + m'_c[c^*C]}{k'_c + m_c}}{m'_c + l'_c + v} = \frac{m_c k_c cC + m_c m'_c[c^*C]}{(m'_c + l'_c + v)(k'_c + m_c)}$$

$$\left(1 - \frac{m_c m'_c}{(m'_c + l'_c + v)(k'_c + m_c)}\right)[c^*C] = \frac{m_c k_c cC}{(m'_c + l'_c + v)(k'_c + m_c)}$$

$$\frac{(m'_c + l'_c + v)(k'_c + m_c) - m_c m'_c}{(m'_c + l'_c + v)(k'_c + m_c)}[c^*C] = \frac{m_c k_c cC}{(m'_c + l'_c + v)(k'_c + m_c)}$$

$$[c^*C] = \frac{m_c k_c cC}{(m'_c + l'_c + v)(k'_c + m_c) - m_c m'_c}$$

So the correct amino acid rate is

$$v[c^*C] = \frac{v m_c k_c cC}{(m'_c + l'_c + v)(k'_c + m_c) - m_c m'_c}$$

b)

Error rate of incorporation

$$Error\ rate = \frac{wrong\ amino\ acid\ rate}{correct\ amino\ acid\ rate} = \frac{v[w^*C]}{v[c^*C]} = \frac{[w^*C]}{[c^*C]}$$

As wrong intermediate concentration $[w^*C]$ can also be derived using the same method as in a) with substituting constants of the correct incorporations into wrong ones,

$$[w^*C] = \frac{m_w k_w wC}{(m'_w + l'_w + v)(k'_w + m_w) - m_w m'_w}$$

Hence,

$$Error\ rate = \frac{m_w k_w wC((m'_c + l'_c + v)(k'_c + m_c) - m_c m'_c)}{m_c k_c cC((m'_w + l'_w + v)(k'_w + m_w) - m_w m'_w)}$$